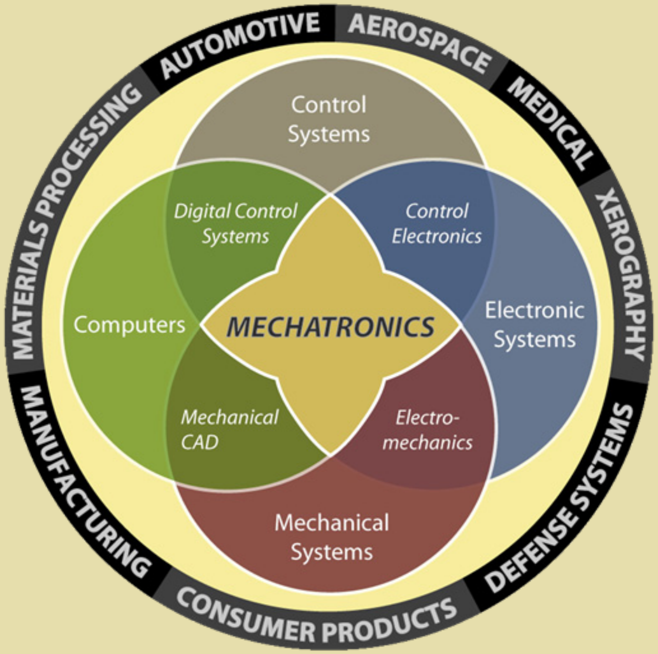
Electrical-Mechanical Analogy



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Electrical – Mechanical Analogies

- A signal, element, or system which exhibits mathematical behavior identical to that of another, but physically different, signal, element, or system is called an analogous quantity or analog.
- Analogous quantities:

force	$\Leftrightarrow$	voltage
velocity	$\Leftrightarrow$	current
displacement	$\Leftrightarrow$	charge
damper	$\Leftrightarrow$	resistor
spring	$\Leftrightarrow$	capacitor
mass	$\Leftrightarrow$	inductor

- Force causes velocity, just as voltage causes current.
- A damper dissipates mechanical energy into heat, just as a resistor dissipates electrical energy into heat.
- Springs and masses store energy in two different ways (potential energy and kinetic energy), just as capacitors and inductors store energy in two different ways (electric field and magnetic field).

Spring Potential Energy

$$\frac{1}{2}Kx^2 = \frac{1}{2}\frac{(Kx)^2}{K} = \frac{1}{2}\frac{f^2}{K} \Leftrightarrow \frac{1}{2}Ce^2 = \frac{1}{2}\frac{q^2}{C}$$

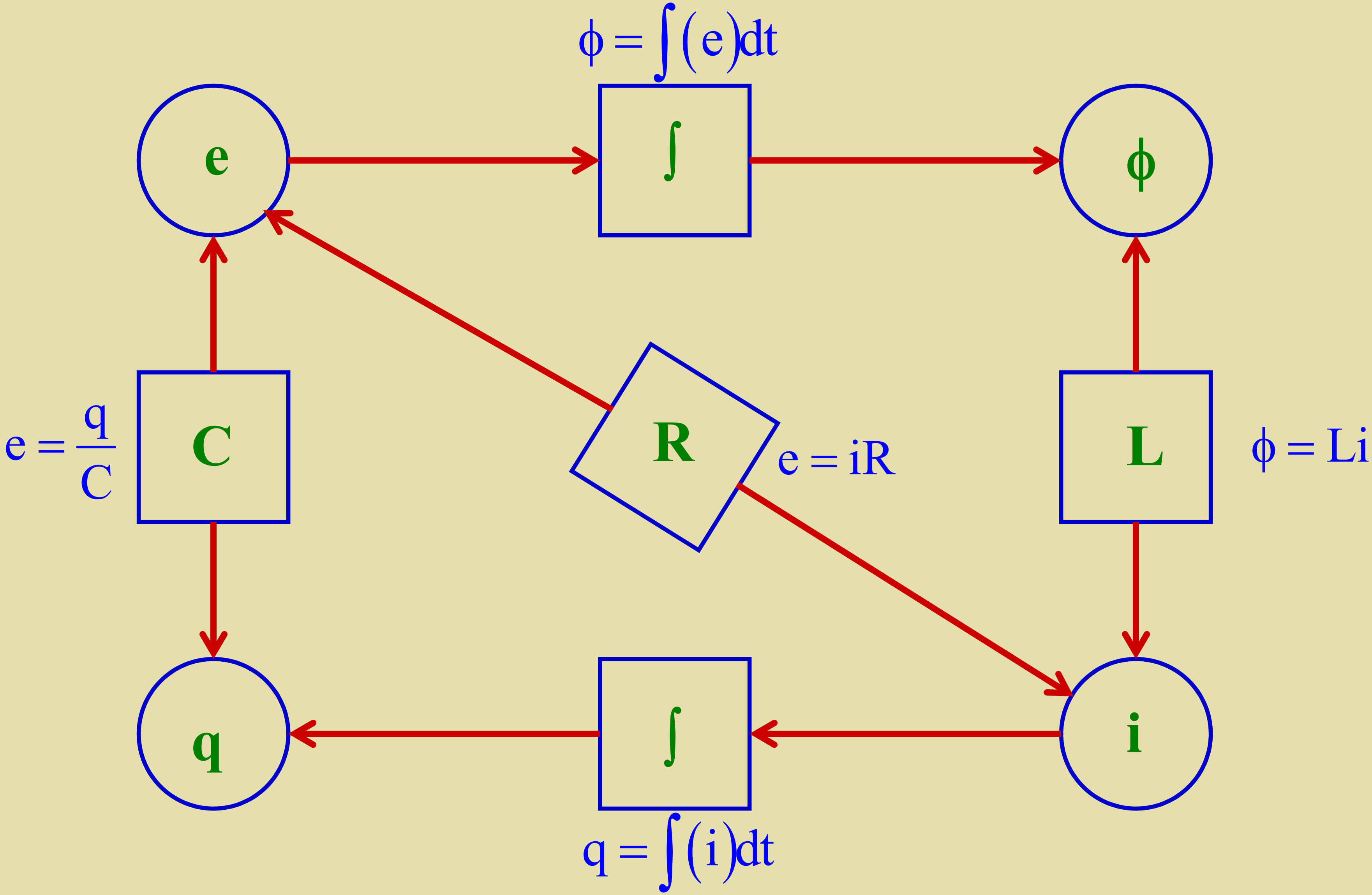
Capacitor Electric Field Energy

Mass Kinetic Energy

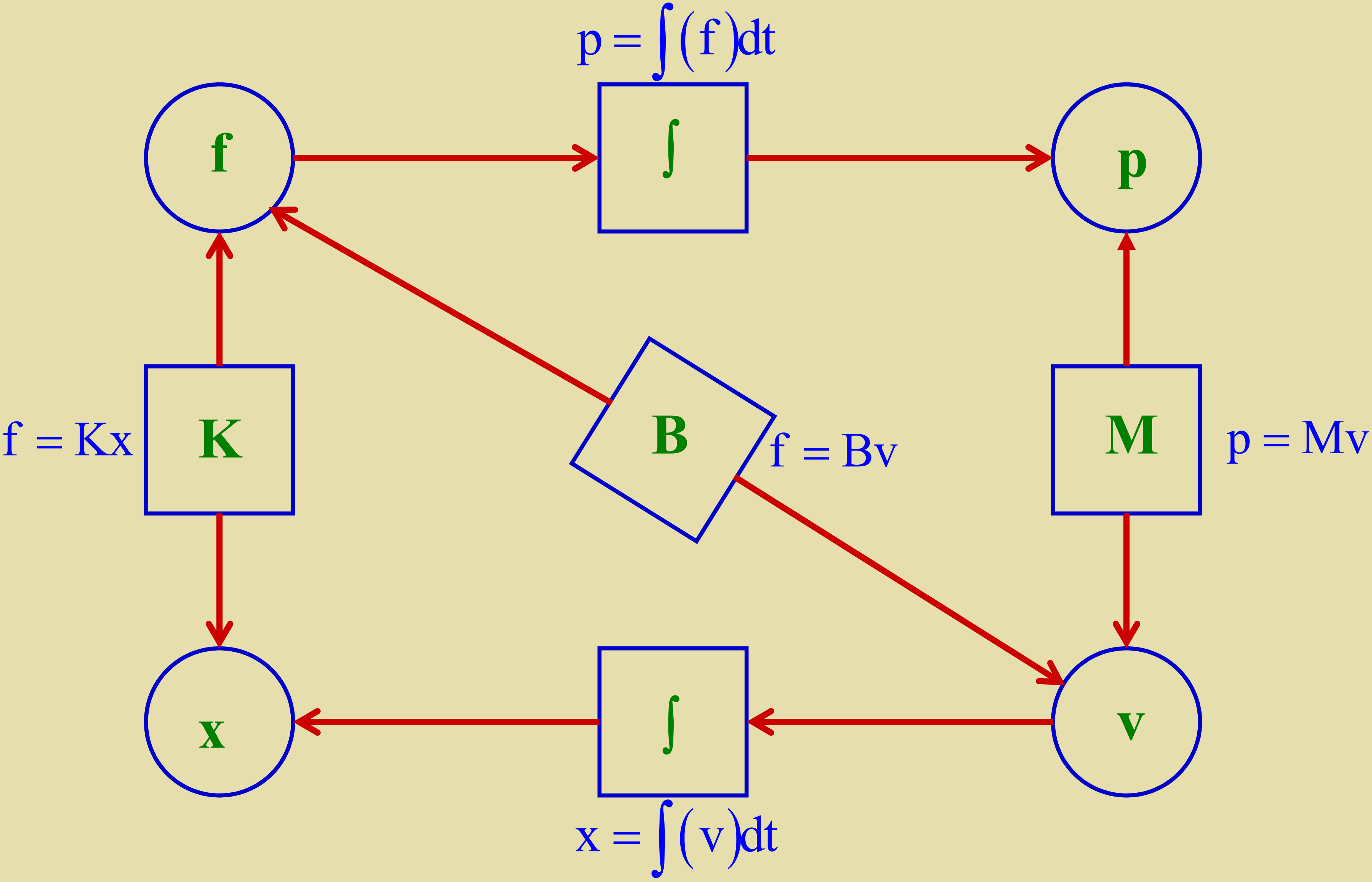
$$\frac{1}{2}Mv^2 \Leftrightarrow \frac{1}{2}Li^2$$

Inductor Magnetic Field Energy

- The product (f)(v) represents instantaneous mechanical power, just as (e)(i) represents instantaneous electrical power.



General Model Structure for Electrical Systems



General Model Structure for Mechanical Systems

Electrical – Mechanical Analogies

force f $\Leftrightarrow$ voltage e

velocity v $\Leftrightarrow$ current i

damper B $\Leftrightarrow$ resistor R

spring K $\Leftrightarrow$ capacitor 1/C

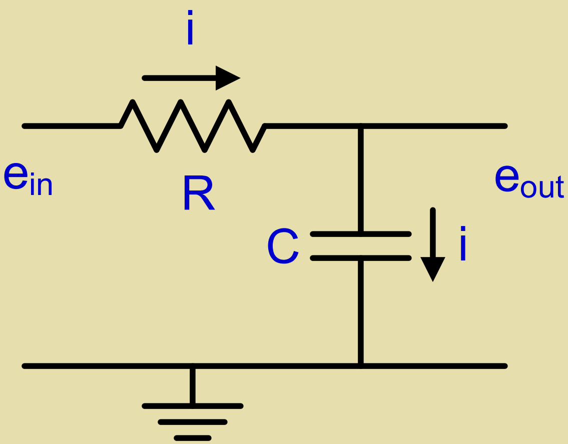
mass M $\Leftrightarrow$ inductor L

$$\text{Resistor } e = Ri \quad \Leftrightarrow \quad \text{Damper } f = Bv$$

$$\text{Inductor } e = L\frac{di}{dt} \quad \Leftrightarrow \quad \text{Mass } f = M\frac{dv}{dt}$$

$$\text{Capacitor } e = \frac{1}{C}\int i\,dt \quad \Leftrightarrow \quad \text{Spring } f = K\int v\,dt$$

RC Electrical System



$$e_{in} - e_R - e_C = 0$$

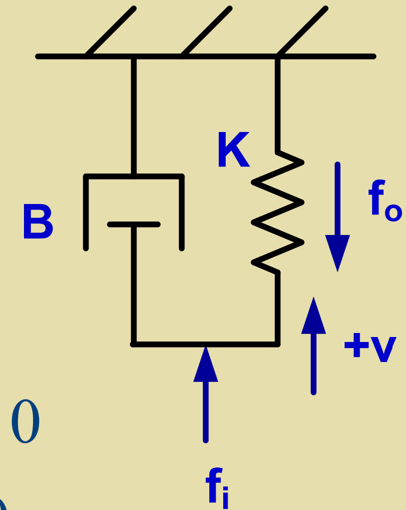
$$e_{in} - iR - e_{out} = 0$$

$$e_{in} - \left(C \frac{de_{out}}{dt} \right) R - e_{out} = 0$$

$$RC \frac{de_{out}}{dt} + e_{out} = e_{in}$$

$$\frac{e_{out}}{e_{in}} = \frac{1}{RCD + 1} \qquad \tau = RC$$

Spring-Damper Mechanical System



$$f_i - f_B - f_K = 0$$

$$f_i - Bv - Kx = 0$$

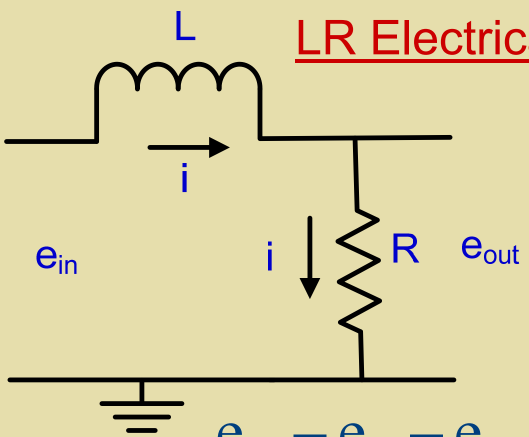
$$f_i - Bv - f_o = 0$$

$$f_i - B\left(\frac{\dot{f}_o}{K} \right) - f_o = 0$$

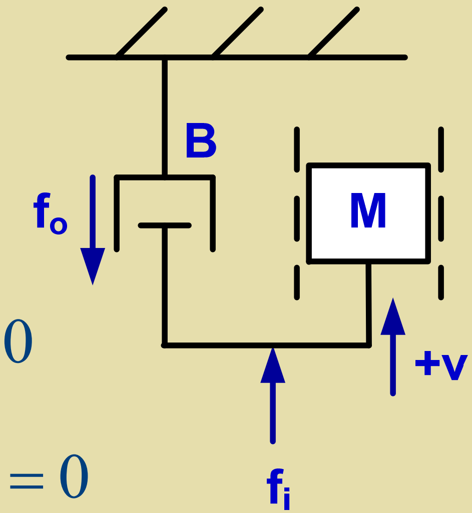
$$\frac{B}{K} \dot{f}_o + f_o = f_i$$

$$\frac{f_o}{f_i} = \frac{1}{\frac{B}{K} D + 1} \qquad \tau = \frac{B}{K}$$

LR Electrical System



Mass-Damper Mechanical System



$$e_{in} - e_L - e_R = 0$$

$$e_{in} - L\frac{di}{dt} - e_{out} = 0$$

$$e_{in} - L\frac{d}{dt}\left(\frac{e_{out}}{R}\right) - e_{out} = 0$$

$$\frac{L}{R}\frac{de_{out}}{dt} + e_{out} = e_{in}$$

$$\frac{e_{out}}{e_{in}} = \frac{1}{\frac{L}{R}D + 1} \qquad \tau = \frac{L}{R}$$

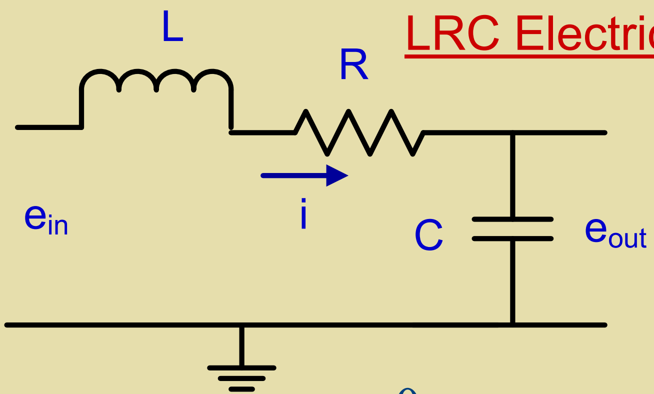
$$f_i - f_B - f_M = 0$$

$$f_i - Bv - M\dot{v} = 0$$

$$f_i - f_o - M\left(\frac{\dot{f}_o}{B}\right) = 0$$

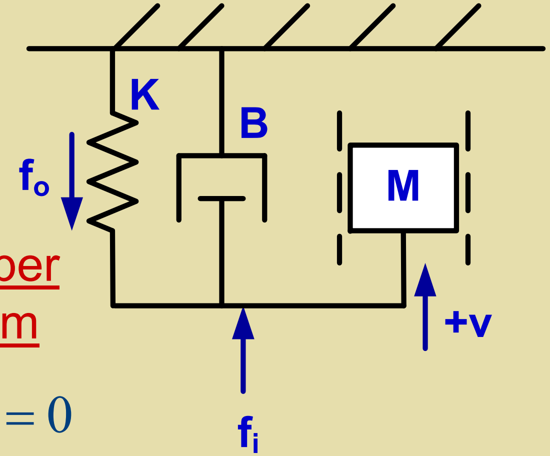
$$\frac{M}{B}\dot{f}_o + f_o = f_i$$

$$\frac{f_o}{f_i} = \frac{1}{\frac{M}{B}D + 1} \qquad \tau = \frac{M}{B}$$



LRC Electrical System

Mass-Spring-Damper Mechanical System



$$e_{in} - e_L - e_R - e_C = 0$$

$$e_{in} - L\frac{di}{dt} - Ri - e_{out} = 0$$

$$e_{in} - L\frac{d}{dt}\left(C\frac{de_{out}}{dt}\right) - R\left(C\frac{de_{out}}{dt}\right) - e_{out} = 0$$

$$LC\frac{d^2e_{out}}{dt^2} + RCdt\frac{de_{out}}{dt} + e_{out} = e_{in}$$

$$\frac{e_{out}}{e_{in}} = \frac{1}{LCD^2 + RCD + 1} = \frac{K_S}{\frac{1}{\omega_n^2}D^2 + \frac{2\zeta}{\omega_n}D + 1}$$

$$\omega_n = \sqrt{\frac{1}{LC}} \quad \zeta = \frac{R}{2}\sqrt{\frac{C}{L}} \quad K_S = 1$$

$$f_i - f_K - f_B - f_M = 0$$

$$f_i - Kx - Bv - M\dot{v} = 0$$

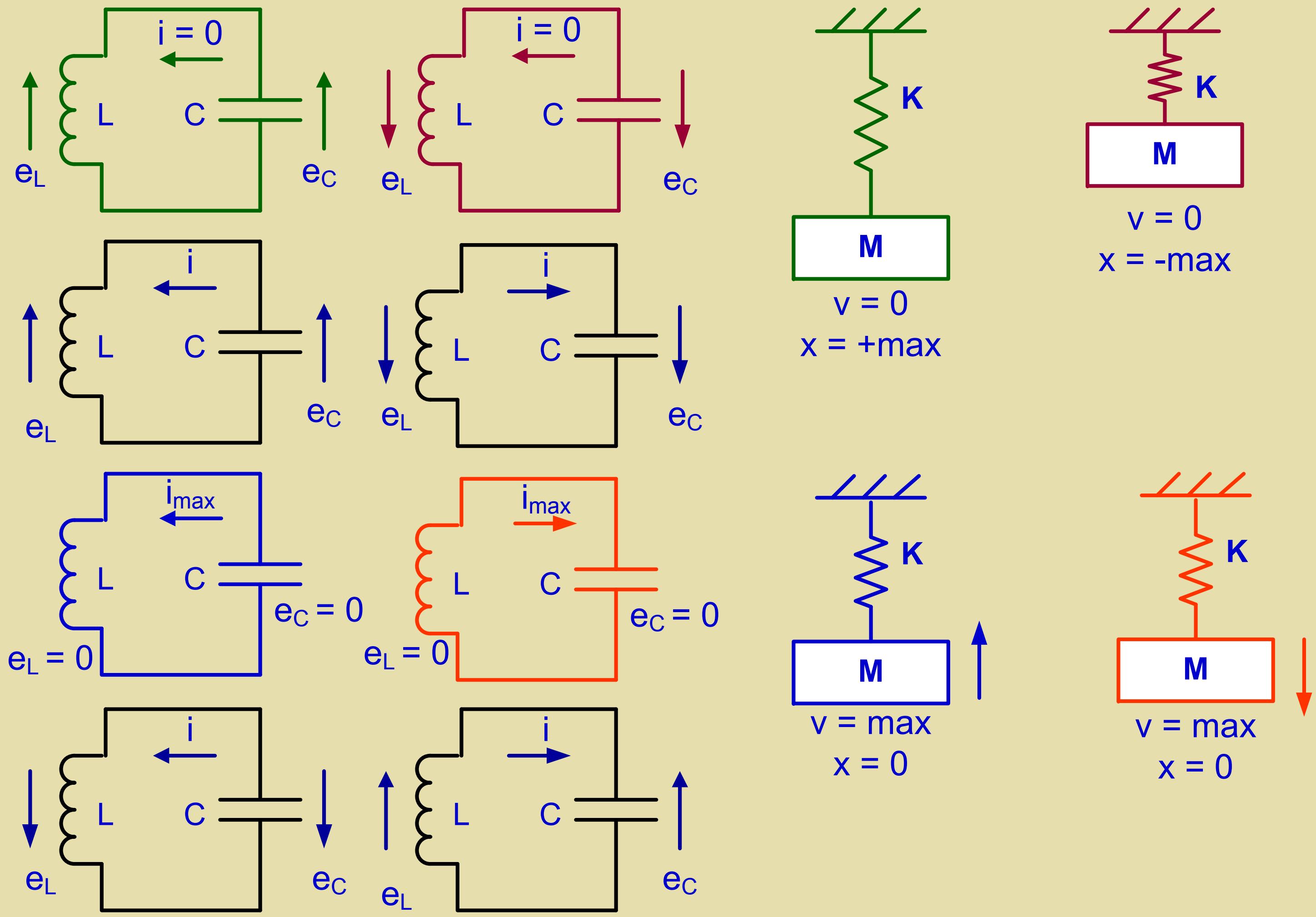
$$f_i - f_o - B\left(\frac{\dot{f}_o}{K}\right) - M\left(\frac{\ddot{f}_o}{K}\right) = 0$$

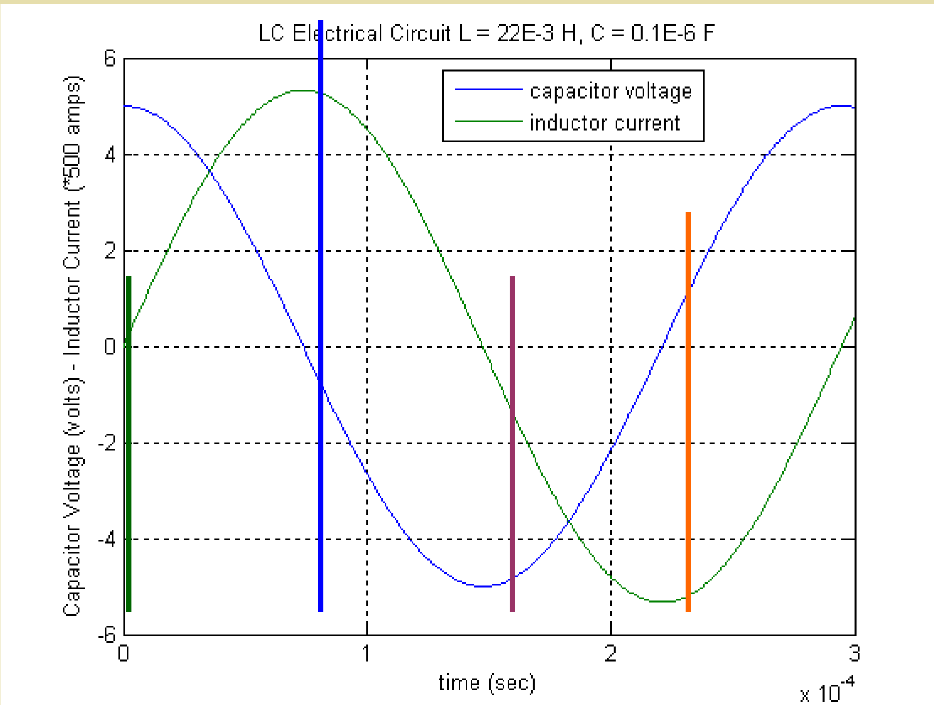
$$\frac{M}{K}\ddot{f}_o + \frac{B}{K}\dot{f}_o + f_o = f_i$$

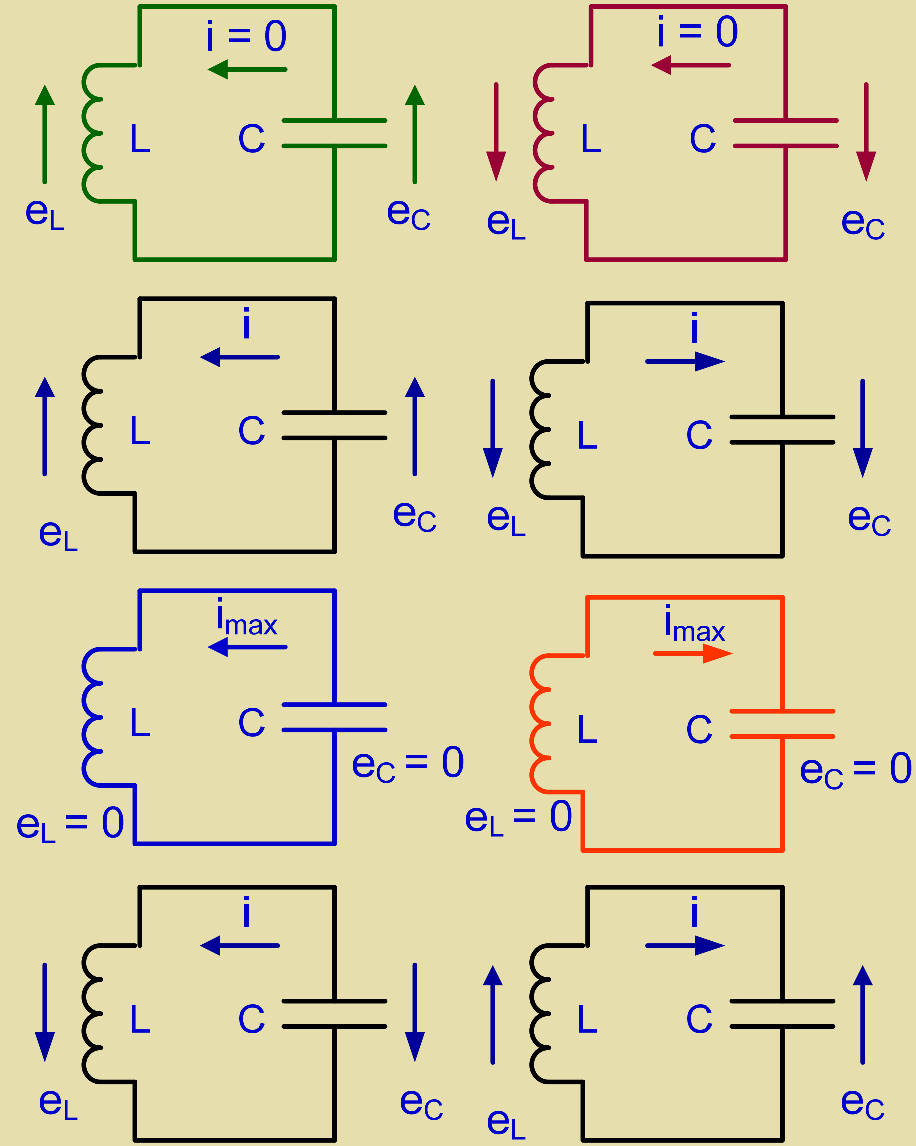
$$\frac{f_o}{f_i} = \frac{1}{\frac{M}{K}D^2 + \frac{B}{K}D + 1} = \frac{K_S}{\frac{1}{\omega_n^2}D^2 + \frac{2\zeta}{\omega_n}D + 1}$$

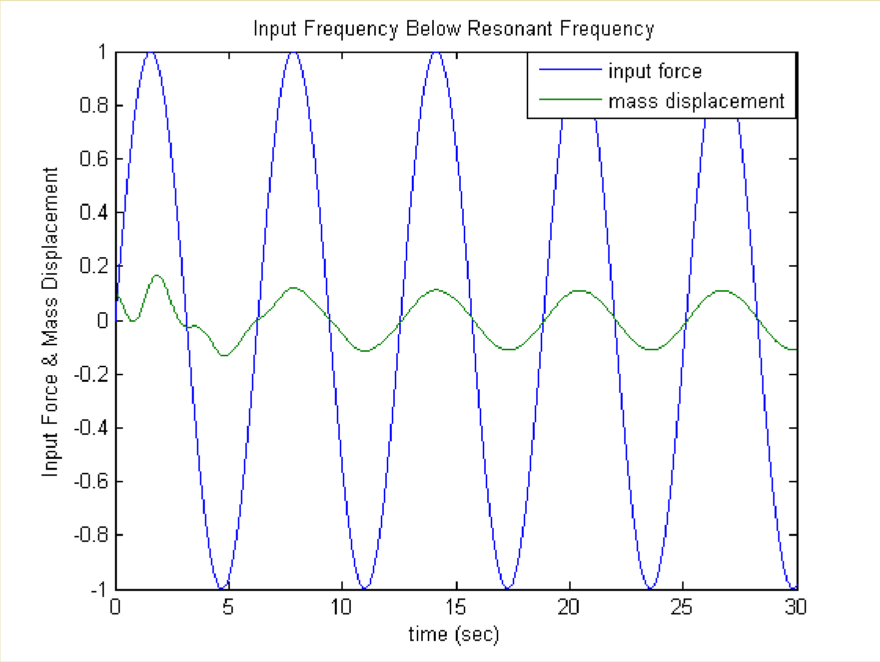
$$\omega_n = \sqrt{\frac{K}{M}} \quad \zeta = \frac{B}{2}\sqrt{\frac{1}{KM}} \quad K_S = 1$$

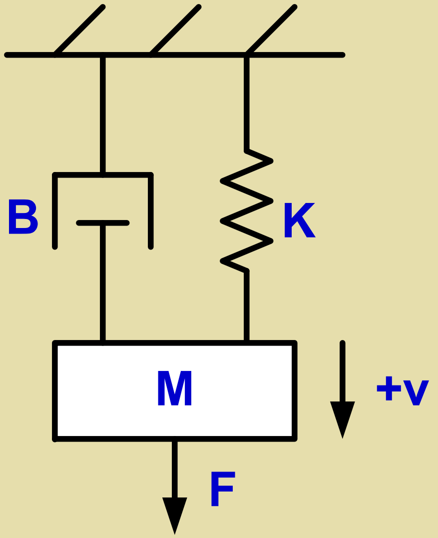
- <u>Inductor-Capacitor (LC) ↔ Mass-Spring (MK) Oscillations</u>

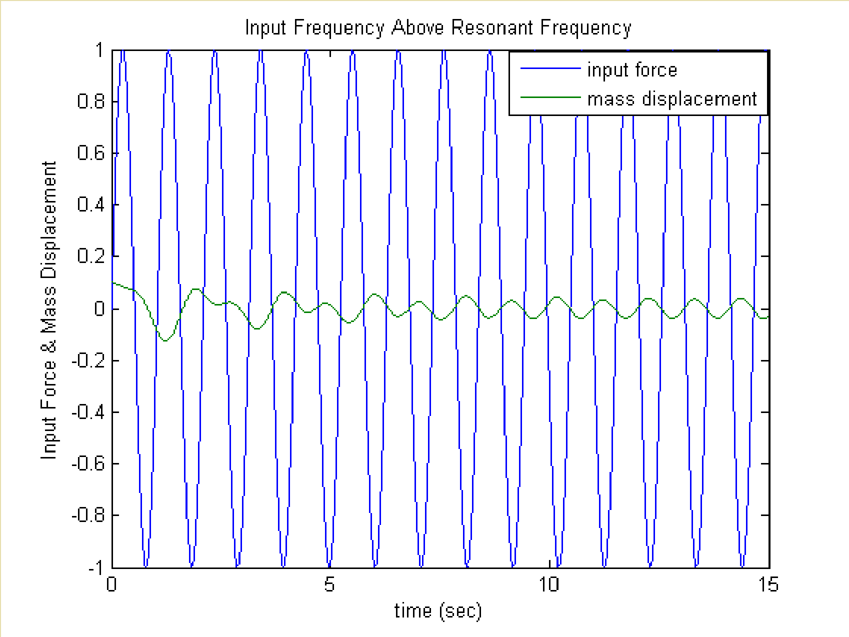



LC Electrical Circuit L = 22E-3 H, C = 0.1E-6 F
capacitor voltage
inductor current
Capacitor Voltage (volts) - Inductor Current (*500 amps)
time (sec)
x 10-4
6
4
2
0
-2
-4
-6
0
1
2
3

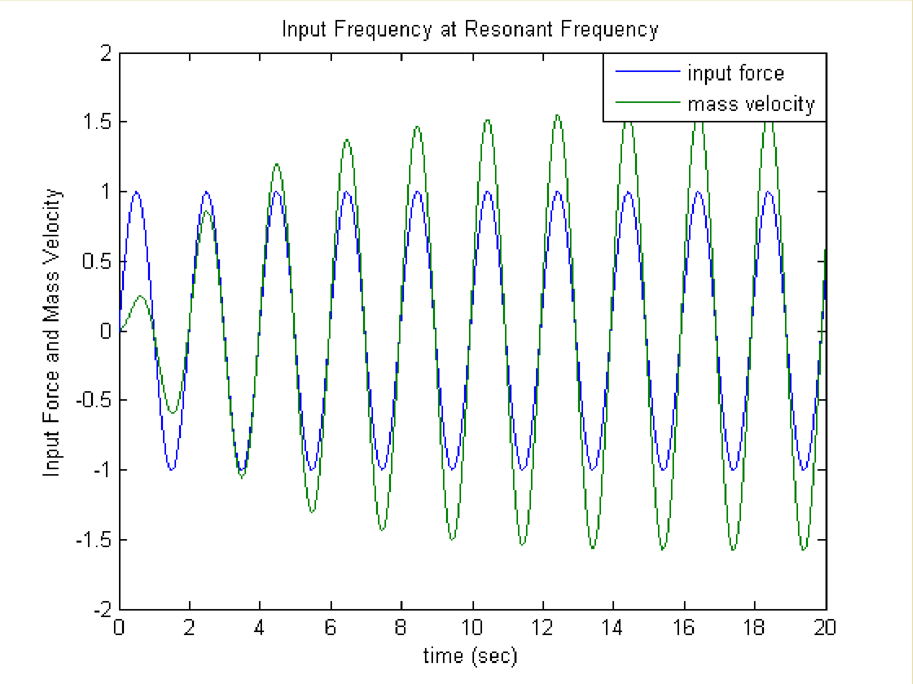

i = 0
L
C
eL
eC
i = 0
L
C
eL
eC
i
L
C
eL
eC
i
L
C
eL
eC
imax
L
C
eL = 0
eC = 0
imax
L
C
eL = 0
eC = 0
i
L
C
eL
eC
i
L
C
eL
eC

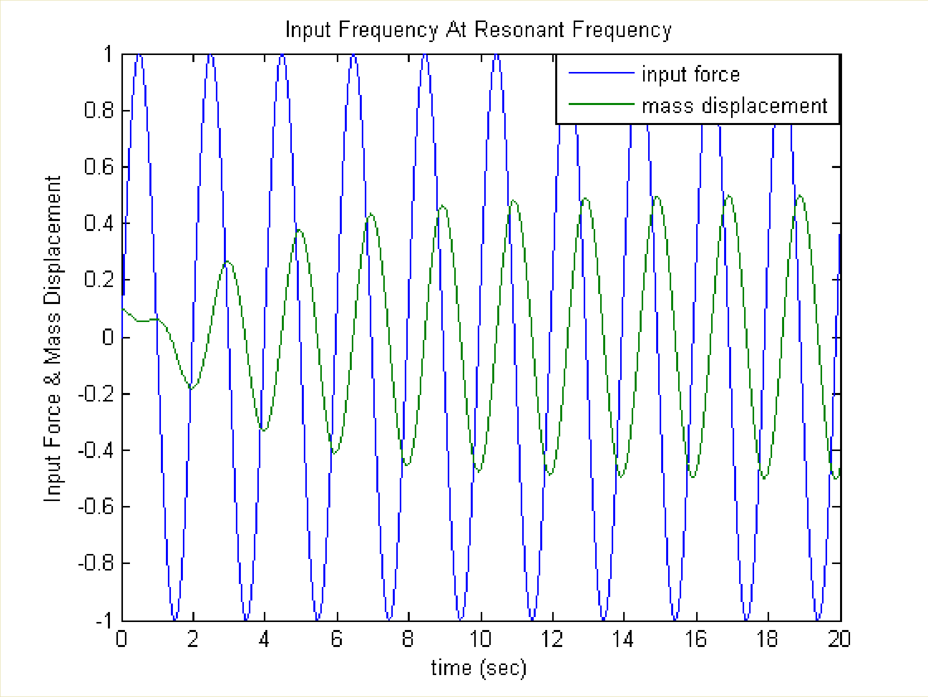

Input Frequency Below Resonant Frequency
input force
mass displacement
Input Force & Mass Displacement
time (sec)


B
K
M
+v
F


Input Frequency Above Resonant Frequency
input force
mass displacement
Input Force & Mass Displacement
time (sec)

Resonance


Input Frequency at Resonant Frequency
input force
mass velocity
Input Force and Mass Velocity
time (sec)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
2
4
6
8
10
12
14
16
18
20


Input Frequency At Resonant Frequency
input force
mass displacement
Input Force & Mass Displacement
time (sec)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
2
4
6
8
10
12
14
16
18
20